Property testing for pixel images
We study $n \times n$ images of white and black pixels in the property testing framework of [1, 2].

Image graph: vertices = black pixels
vertices $(i, j)$ and $(i', j')$ are connected by an edge if $|i - i'| + |j - j'| = 1$
An image is connected if its image graph is connected.

- An image is $\epsilon$-far from connected if at least $\epsilon n^2$ pixels need to change it to make it connected.
- A tester should accept connected images and reject images that are $\epsilon$-far from connected.
- A tester has $1$-sided error if it always accepts connected images.
- A tester is nonadaptive if it prepares all queries in advance; otherwise, the tester is adaptive.

Nonadaptive

Adaptive

Lower bound
Any connectedness tester makes $\Omega \left( \frac{1}{\epsilon^2 \log \frac{1}{\epsilon}} \right)$ queries.

Goal:
- Create a family of $\epsilon$-far images for which it is hard to catch a proof of disconnectedness

High level idea:
- Create subimages of different sizes of a particular template and hide it in the image
  - If the subimage is small, it is easy to test connectedness, but hard to find the object
  - If the subimage is big, it is easy to find, but need many queries to prove disconnectedness
- Important property: if the tester invests in searching for an object of a specific size, it doesn’t help too much for other sizes

Construction

- We pick a position and a size and place a checkerboard, so that we have $\epsilon n^2$ components. We pick the size from $\Theta \left( \log \frac{1}{\epsilon} \right)$ potential levels in power of 2 increments.
- In the white spaces, we create some bridges that connect two consecutive black checkerboard squares
- For a given position and level, an algorithm needs to see a constant fraction of pixels in the white cell to confirm all bridges are disconnected.
- Overall, a tester must query $\Omega \left( \frac{1}{\epsilon^2 \log \frac{1}{\epsilon}} \right)$ pixels per level
- The information gathered from other levels is not enough to make the total lower bound complexity lower than $\Omega \left( \frac{1}{\epsilon^2 \log \frac{1}{\epsilon}} \right)$

Open Problems
- Adaptive lower bound
- Tight bounds

Adaptive lower bound

Tight bounds

Reference

Analysis
- We always accept connected images
- We reject an image that is $\epsilon$-far from connected with probability $\geq 2/3$